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THE EFFECT OF MASS TRANSFER ON RECOVERY FACTORS IN LAMINAR BOUNDARY LAYER FLOWS*

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Abstract—Exact numerical solutions for adiabatic, self similar boundary layer flows have been obtained in a comprehensive parametric study. The problem parameters include: pressure gradient (β = separation to 2·0) flow kinetic energy ($E = u_e^2/2H_e$ = 0·5, 0·7, 0·9, 0·95) and surface mass transfer rate ($-f_s = -10{\cdot}0$ to 1·0). In order to exhibit clearly the effect of gas properties on the adiabatic wall enthalpy and the recovery factor $r = (h_{ad} - h_e)/(u_e^2/2)$, attention is directed primarily at model gases with constant Prandtl numbers (Pr = 0·6, 0·74, 0·9), density inversely proportional to the static enthalpy, and a power law for viscosity ($\mu \propto h^{\omega}$; ω = 0·5, 0·7, 1·0). Relevance of the model gas flows to real gas flows in engineering problems is demonstrated by the inclusion of air, N_2 and CO_2 data, which follow closely the model gas results. It is found that the recovery factor decreases monotonically with increasing β, the effect being most pronounced at high injection rates. Also the effectiveness of injection in reducing the recovery factor increases with decreasing ω, and the deviation of r from unity increases with increasing Mach number. Upon normalization with $Pr_s^{\frac{1}{2}}$ the recovery factors for real gases correlate well with the ω = 0·5 model gas results. The data presented finds its primary utility in the engineering calculation of ablative and transpiration cooling.

NOMENCLATURE

C, $\rho\mu/\rho_e\mu_e$, normalized density–viscosity product;
C_p, heat capacity;
E, $u_e^2/2H_e$, flow kinetic energy, or Mach number, parameter;
f, dimensionless stream function;
g, H/H_e;
H, total enthalpy;
h, enthalpy;
k, thermal conductivity;
$\dot{m}$, mass transfer rate;
Pr, Prandtl number;
R, radius of curvature;
r, recovery factor;
T, absolute temperature;
s, y, boundary layer coordinates;
u, v, velocity components;
β^0, $2\mathrm{d}\ln u_e/\mathrm{d}\ln\xi$, free stream acceleration parameter;
β, $\beta^0/(1 - E)$, pressure gradient parameter;
δ_1, displacement thickness;
θ, momentum thickness;
θ_c, cone half-angle;
ε, geometrical index;
η, transformed coordinate normal to the surface;
μ, dynamic viscosity;
ρ, density;

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ξ, transformed coordinate along the surface;
τ_s, surface shear stress;
ψ, stream function;
ω. exponent in viscosity–enthalpy relation.

Subscripts
ad, condition at an adiabatic wall;
e, free stream;
r, evaluated at the Eckert reference condition;
s, surface.

Superscripts
$'$, differentiation with respect to η;
$*$, zero mass transfer.

INTRODUCTION

THE ADIABATIC wall enthalpy, and thus the recovery factor $r = (h_{ad} - h_e)/(u_e^2/2)$, are both of direct engineering interest and find utility in the correlation of heat transfer data for high Mach number flows. The effects of both wall injection and suction are of current interest. For example, the widespread use of ablative heat shields and transpiration cooling to protect surfaces from high speed flows indicates a need for studies of the effect of injection on the recovery factor. To effect boundary layer control, suction is applied to the inlet duct surfaces of high performance aircraft, and is also being considered for the inlets of hypersonic air-breathing vehicles. Extensive data are available for the air boundary layer with zero pressure gradient and no surface mass transfer. For flows with pressure gradients, and for mass transfer, comprehensive data do not appear in the literature, e.g. there are no data computed with realistic gas properties for decelerating flows or flows with suction. Inclusion of the pressure gradient as a primary problem parameter is particularly necessary, owing to the complex geometries characterizing aircraft, and hypersonic vehicles such as space shuttles.

Previous work may be summarized briefly as follows. For zero pressure gradient flows over impermeable surfaces, Eckert [1], in 1952, reviewed the available data. He showed that the adiabatic wall temperature was best expressed in the form of a recovery factor r, defined by

$$r = \frac{T_{ad} - T_e}{u_e^2/2C_p} \tag{1}$$

and found that for air flows, with both model and real gas properties, the recovery factor was well correlated by the simple expression

$$r = \sqrt{Pr} \tag{2}$$

provided properties (C_p and Pr) were evaluated at the Eckert reference temperature T_r, given by

$$T_r = T_e + 0{\cdot}58(T_s - T_e) + 0{\cdot}19(T_{ad} - T_e). \tag{3}$$

More recently the practice has been to use enthalpy rather than temperature, leading to alternative forms of equations (1) and (3). For example, Kays [2] gives

$$r = \frac{h_{ad} - h_e}{u_e^2/2} \tag{4}$$

$$h_r = h_e + 0{\cdot}50(h_s - h_e) + 0{\cdot}22(h_{ad} - h_e) \tag{5}$$

where the constants in equation (5) are taken from a more recent publication of Eckert [3]. For self-similar flows over impermeable surfaces the effect of pressure gradient on the recovery factor was investigated by Tifford and Chu [4] under the assumption of constant properties. Values of r were computed in a range of the pressure gradient parameter β^0 from $-0{\cdot}1988$ (claimed separation) to 2·0. For $Pr = 0{\cdot}7$ the variation with β^0 was found to be less than 1·5 per cent, and r exhibited a maximum near $\beta^0 = 0$. More recently, Rogers [5] computed r for compressible flows under the unrealistic assumption of a linear viscosity–temperature relation ($\omega = 1{\cdot}0$), constant specific heat, $Pr = 0{\cdot}723$, and the hypersonic limit ($E = 1{\cdot}0$). Rogers focused primarily on flows subjected to adverse pressure gradients and, in contrast to Tifford and Chu, found that r has its maximum value at separation, and decreases monotonically with β.

For compressible flows over permeable surfaces, with injection, the most comprehensive computations are those of Dewey and Gross [6, 7] who considered a model gas with properties $\rho \propto h$, $\mu \propto h^{\omega}$, and Pr = constant. However they did not obtain data for suction or negative values of β, and, for $\beta > 1{\cdot}0$ only four data points were computed, viz. $\beta = 1{\cdot}4$, $E = 0{\cdot}7$; $\beta = 1{\cdot}5$, $E = 0{\cdot}25$, $0{\cdot}5$; $\beta = 2{\cdot}0$, $E = 0{\cdot}25$, all for zero mass transfer. We note in passing that the calculation of laminar boundary layers becomes more difficult as β and E are increased. The objective of the present study was to compute data for r sufficient to allow a systematic assessment of the effects on r, of Prandtl number, the viscosity law exponent ω, the pressure gradient parameter β, the kinetic energy E, and the injection parameter $-f_s$. In addition the applicability of model gas results to real gas flows was determined.

ANALYSIS

A curvilinear, orthogonal coordinate system is chosen such that s is measured along, and y perpendicular to, the surface; the corresponding velocity components are u and v respectively. For steady laminar boundary layer flow the governing equations are

mass:

$$\frac{\partial}{\partial s}(\rho u R^{\varepsilon}) + \frac{\partial}{\partial y}(\rho v R^{\varepsilon}) = 0 \tag{6}$$

momentum:

$$\rho u \frac{\partial u}{\partial s} + \rho v \frac{\partial u}{\partial y} = -\frac{\mathrm{d}p}{\mathrm{d}s} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \tag{7}$$

total enthalpy:

$$\rho u \frac{\partial H}{\partial s} + \rho v \frac{\partial H}{\partial y} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial y}\left(\mu u \frac{\partial u}{\partial y}\right) \tag{8}$$

where, in the case of a real gas, k is the total thermal conductivity. The geometrical index ε assumes a value of 0 for planar flows and 1 for axisymmetric ones. Second order boundary layer effects, such as transverse curvature, will be ignored. Body forces and radiative energy transport are absent. The boundary conditions imposed on the set of equations are

$$y = 0: \quad u = 0, \quad \rho v = \dot{m}, \quad \partial T/\partial y = 0 \tag{9}$$

$$y \to \infty: u \to u_e, \quad H \to H_e. \tag{10}$$

Following Lees [8], the transformation $x,y \to \xi, \eta$ is made where the Levy and Mangler transformations have been combined in defining

$$\eta = \frac{\rho_e u_e}{(2\xi)^{\frac{1}{2}}} \int_0^y R^{\varepsilon} \frac{\rho}{\rho_e} \mathrm{d}y \tag{11}$$

$$\xi = \int_0^s \rho_e \mu_e u_e R^{2\varepsilon} \mathrm{d}s. \tag{12}$$

A stream function ψ is introduced such that

$$\rho u R^{\varepsilon} = \frac{\partial \psi}{\partial y}; \quad \rho v R^{\varepsilon} = -\frac{\partial \psi}{\partial s} \tag{13}$$

and choosing

$$\psi(\xi, \eta) = (2\xi)^{\frac{1}{2}} f(\eta)$$

leads to

$$\frac{u}{u_e} = \frac{\partial f}{\partial \eta} \equiv f'.$$

Under this transformation, the conservation equations for self-similar flows become

momentum:

$$(Cf'')' + ff'' = \beta^0\left(f'^2 - \frac{\rho_e}{\rho}\right) \tag{14}$$

total enthalpy:

$$\left(\frac{C}{Pr} g'\right)' + fg' = E\left[2C\left(\frac{1}{Pr} - 1\right)f'f''\right]' \tag{15}$$

subject to the boundary conditions

$$\eta = 0: f' = 0, f = f_s, g' = 0 \tag{16}$$

$$\eta \to \infty: f' \to 1, g \to 1 \tag{17}$$

where

$$C = \frac{\rho\mu}{\rho_e \mu_e}; \quad \beta^0 = 2\frac{\mathrm{d}\ln u_e}{\mathrm{d}\ln\xi}; \quad g = \frac{H}{H_e};$$

$$E = \frac{u_e^2}{2H_e}.$$

Following Dewey and Gross [7], the model gas is chosen to have the properties,

$$\rho \propto h^{-1}; \mu \propto h^{\omega}; Pr = \text{constant}$$

thereby eliminating enthalpy level as a parameter. For the model gas equation (14) may be recast in the more convenient form

$$(Cf'')' + ff'' = \beta(f'^2 - g). \qquad (18)$$

The parameter $\beta = \beta^0/1 - E$ is found to be more appropriate than β^0 for scaling pressure gradient effects in high Mach number flows. For the model gas the problem parameters are, therefore, β, E, f_s, Pr and ω. A value of the viscosity exponent, ω, equal to 1·0 yields the familiar constant density–viscosity product simplification ($C = 1{\cdot}0$); $\omega = 0{\cdot}7$ is appropriate for low temperature air flows, while $\omega = 0{\cdot}5$ may be regarded as a limiting value for high temperature flows.

Real gas flows were treated using the total properties approach, thereby introducing the additional assumption of an elemental composition invariant across the boundary layer. The required data for ρ, $(\rho\mu)$, and total Prandtl number were taken from N. B. Cohen [9] for air, and from Marvin and Deiwert [10] for N_2 and CO_2. The Clutter and Smith [11] curve fit for the variation of Prandtl number with enthalpy was used in the air flow calculations. (Note that for real gases equation (14) cannot be eliminated in favor of equation (18) and hence the need for tabulations of both ρ and $(\rho\mu)$.)

In solving equations (14) or (18), and equation (15), we are not concerned that most of the solutions obtained do not have exactly corresponding physically real self-similar flows. Our objective is to provide engineering data for use in the local similarity method of computing boundary layers, and to study the effects of various problem parameters, especially β and f_s.

The method of solution employed in this study was developed in Reference [12] and applied to the computation of a wide range of boundary layer problems. Formal integration of equations (15) and (18), subject to the imposed boundary conditions, yields a relation which, in functional form, may be written as

$$X = FX \qquad (19)$$

where F is an operator on the solution vector X and is itself a function of X since it consists of the basic functions of the conservation equations. X is the solution vector whose components are the principal dependent variables of the conservation equations. The solution method is an iterative one. Special care is taken to restrict the range of the operator F which ensures establishment of a converging sequence by forcing successive solutions into a region where the Banach fixed point theorem applies. Weighted averaging of successive solutions satisfactorily restricted the operator range for all the solutions presented in this paper. The analytical nature of the solution made programming a simple matter of organizing the variables and evaluating the required integrals. All the integrals are well behaved functions so that Simpson's rule was sufficiently accurate for the numerical integrations. The number of integration steps across

Table 1. Comparison with Dewey and Gross [7].

β	E	Dewey and Gross [7]		Present study	
		f''_s	g_s	f''_s	g_s
0·0	0·5	0·5671	0·9151	0·5672	0·9151
0·5	0·5	1·074	0·9030	1·074	0·9030
0·5	0·8	1·336	0·8393	1·336	0·8393
0·75	0·5	1·240	0·9004	1·254	0·8994
1·0	0·5	1·412	0·8965	1·410	0·8965
1·5	0·5	1·679	0·8922	1·677	0·8922
2·0	0·25	1·768	0·9452	1·768	0·9452

$Pr = 0{\cdot}7$, $\omega = 0{\cdot}5$, $f_s = 0$

the boundary layer was 151 in all cases. Computer time (IBM 360/91 system) was about 1·3 s per case for four place accuracy. Table 1 shows a comparison of our data with representative samples from Dewey and Gross [7]; the agreement is seen to be excellent.

RESULTS AND DISCUSSION

Table 2. Classification of the flows computed

Prandtl number	ω	E	Mass transfer
0·9	0·5	0·5, 0·7, 0·9, 0·95	Injection
0·9	0·7	0·5, 0·7, 0·9, 0·95	Injection
0·740	0·5	0·5, 0·7, 0·9, 0·95	Injection
0·740	0·7	0·5, 0·7, 0·9, 0·95	Injection
0·740	1·0	0·5, 0·7, 0·9, 0·95	Injection
0·740	0·5	0·5, 0·7, 0·9	Suction
0·740	1·0	0·5, 0·7, 0·9	Suction
0·6	0·5	0·5, 0·7, 0·9, 0·95	Injection
0·6	0·7	0·5, 0·7, 0·9, 0·95	Injection

Table 2 lists the situations that were considered. In all, over 2200 cases were computed. In [12] can be found complete tabulations of the dimensionless adiabatic wall enthalpy g_s, the wall shear stress τ_s, the $(\rho\mu)$ product evaluated at the wall and at the Eckert reference enthalpy, C_s and C_r respectively, the displacement thickness δ_1 and the momentum thickness θ. The pressure gradient parameter ranged from separation to 2·0 while that for the injection parameter f_s was 10 (strong suction) to $-1·0$. The recovery factor was computed from the adiabatic wall enthalpy via the relation

$$r = 1 - \frac{1 - g_s}{E}. \tag{20}$$

Effect of pressure gradient parameter β

Figures 1–3 present data for r as a function of β with f_s as parameter. These results, for selected values of Pr, ω and E, are typical of all the data obtained. The effect of β on r has a marked dependence on mass transfer. At high suction rates, for which r tends to unity, there is no discernible effect whereas the effect is most pronounced at high injection rates. The limiting envelopes, corresponding to pressure gradient induced boundary layer separation, were computed with a special program. This program inverts the usual boundary layer calculation procedure by imposing the condition of zero wall shear stress and then determining the corresponding pressure gradient. In Fig. 4 the recovery factor for separating boundary layers is shown for various values of f_s and E. The curves are close together and tend to the asymptotes $r = 1·0$ (increasing suction) and $\beta = 0$ (increasing blowing). An increasingly high adverse pressure gradient is required for separation as suction is increased; on the other hand the separation adverse pressure gradient tends to zero at large blowing rates. The limiting case of boundary layer "blow-off" on a flat plate cannot be computed exactly using a boundary layer computational technique since in the limit a singular perturbation problem must be considered. The difficulty is that in the Blasius equation, the condition of $f'(0) = f''(0) = 0$ implies that all terms of a Maclaurin expansion will vanish, and no means for adjusting f' to its outer value of 1·0 can be found for any integration scheme. With our

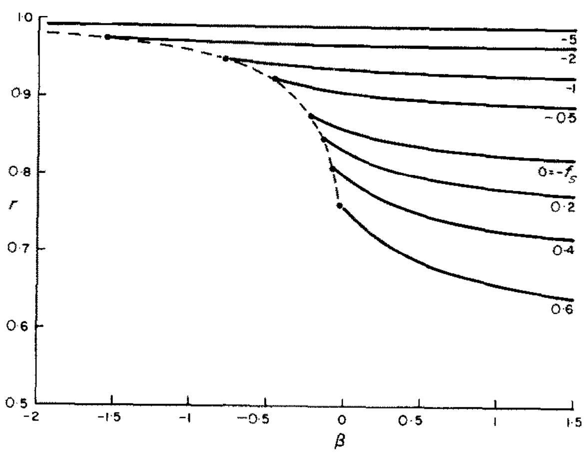

FIG. 1. Variation of recovery factor with pressure gradient parameter β for various injection and suction rates. $Pr = 0·740$; $\omega = 0·5$; $E = 0·5$.

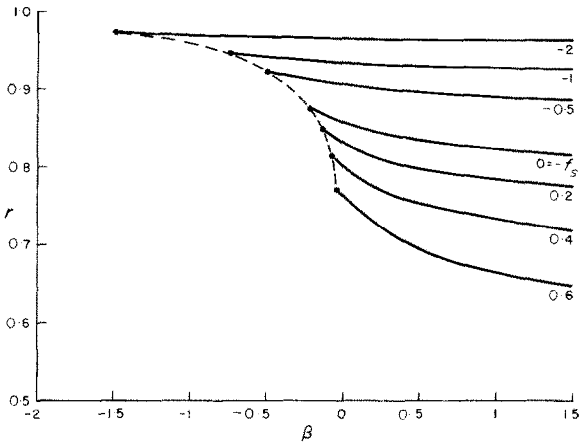

FIG. 2. Variation of recovery factor with pressure gradient parameter β for various injection and suction rates. $Pr = 0{\cdot}740$; $\omega = 0{\cdot}7$; $E = 0{\cdot}5$.

scheme we observe that increasing blowing increases the boundary layer thickness and drives the separation pressure gradient towards zero without ever reaching that value exactly.

Figure 5 presents data for $Pr = 0.740$ and $f_s = 0$ on an expanded ordinate scale. For zero injection the variation of r with β is seen to be remarkably similar for all values of E and ω, r increasing with increasing ω. Figure 5 also

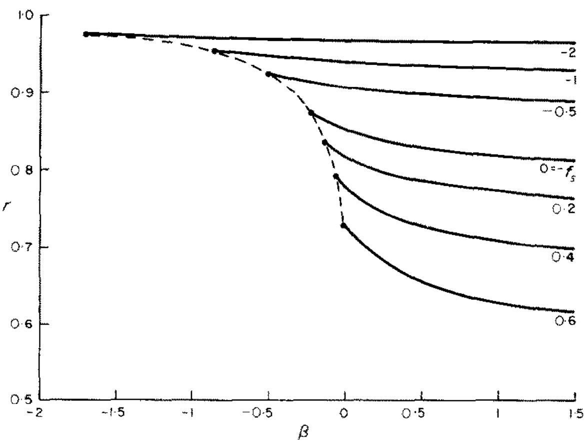

FIG. 3. Variation of recovery factor with pressure gradient parameter β for various injection and suction rates. $Pr = 0{\cdot}740$; $\omega = 0{\cdot}7$; $E = 0{\cdot}9$.

demonstrates that, for zero injection, r is practically independent of E for $\omega = 1{\cdot}0$ (i.e. $C = 1{\cdot}0$, a linear viscosity–enthalpy relation). A number of authors who have computed data for high speed flows, under the assumption of $C = 1{\cdot}0$, have apparently been unaware of this fact. For example, the data of Rogers [5] which was computed for the hypersonic limit ($E = 1{\cdot}0$), does, in one sense, have more general applicability. On the other hand, the effect of E for $C \neq 1{\cdot}0$ displayed in Fig. 5, shows that $C = 1{\cdot}0$ is an unrealistic oversimplification in high speed flows.

We observe that, in accord with the data of Rogers [5], r is a monotonic decreasing function of β in the range studied. Since this result is in apparent contradiction with that obtained by Tifford and Chu [4] cited previously, we also computed the recovery factor for a truly constant property flow ($C = 1{\cdot}0$, ρ = constant). It might be noted in passing that Tifford and Chu obtained the result that the recovery factor was not equal to unity for Prandtl number of one. The energy equation for $Pr = 1{\cdot}0$ and $C = 1{\cdot}0$ is

$$g'' + fg' = 0. \tag{21}$$

Straightforward integration by means of an integrating factor yields the result

$$g = K_1 + K_2 \int_0^\eta \exp\left(-\int_0^\eta f \mathrm{d}\eta\right) \mathrm{d}\eta \tag{22}$$

which, with the boundary conditions $g'(0) = 0$ and $g \to 1$ as $\eta \to \infty$, leads to the conclusion

$$g \equiv 1{\cdot}0. \tag{23}$$

The failure of Tifford and Chu to observe this simple analytical result is surprising, and their calculated results of $r > 1{\cdot}0$ for $Pr = 1{\cdot}0$ are unacceptable. Our data are compared with the results of Tifford and Chu in Table 3, where it can be seen that Tifford and Chu are in error and r is a monotonically decreasing function of β for constant property flows as well. By means of the same integrating factor as was employed to obtain equation (22), the energy equation may be

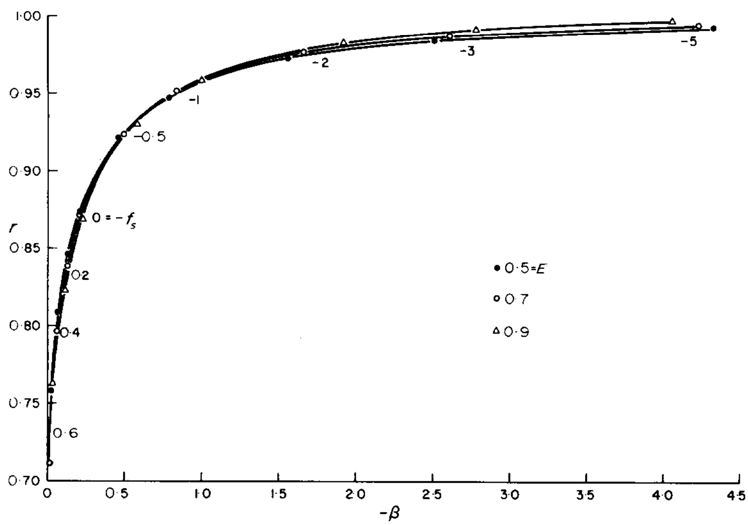

FIG. 4. Envelopes of separation recovery factors. $Pr = 0{\cdot}740$, $\omega = 0{\cdot}5$.

integrated formally and straightforward arguments indicate that the recovery factor must increase as separation is approached. This result is also in contradiction to the findings of Levy and Seban [13], who in a discussion of their own computed data for constant property flows, state that a decrease in recovery factor for separation flow is to be expected in terms of movement of the region of maximum dissipation away from the wall. This argument is physically

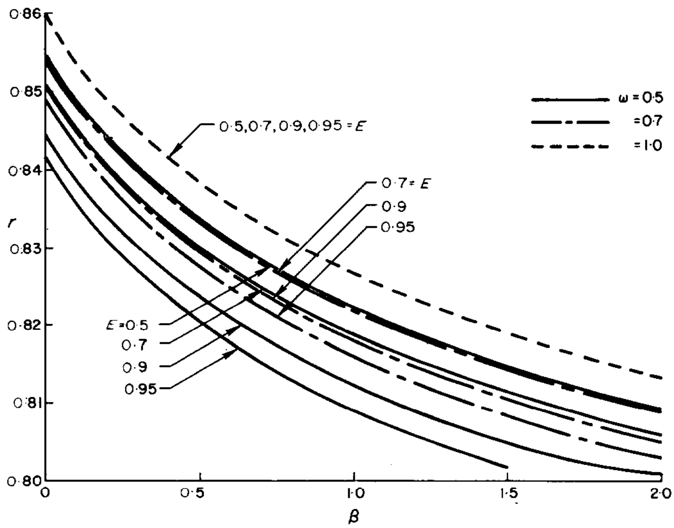

FIG. 5. Zero mass transfer recovery factor as a function of pressure gradient parameter β. $Pr = 0{\cdot}740$.

unsound as it fails to take into account the fact that convection normal to the wall tends to disappear in separating flows.

Table 3. Comparison with constant property data of Tifford and Chu [4]

β^0	r Tifford and Chu [4]	r Present study
−0·1988	0·844	0·8561
−0·19		0·8548
−0·18		0·8534
−0·15		0·8495
−0·14	0·850	0·8483
−0·10		0·8441
0	0·846	0·8357
0·5	0·844	0·8114
1·0	0·840	0·7983

$Pr = 0{\cdot}7, \rho/\rho_e = 1{\cdot}0, C = 1{\cdot}0$

We have established that r decreases monotonically with β in the range $\beta_{\text{separation}} \leqslant \beta \leqslant 2{\cdot}0$. Furthermore, the curves in Fig. 5 indicate little reason to believe that this monotonic behavior will not persist for values of β appreciably in excess of 2·0. Now Dewey and Gross [7] state that, although they have not been able to prove it analytically, it appears that $r = 1{\cdot}0$ for all values of Pr, ω and E in the limit $\beta \to \infty$. In the light of our data this assertion is surprising since it requires that r go through a minimum at a value of β greater than 2·0. Furthermore, on physical grounds, it is very difficult to accept that the recovery factor could be independent of Prandtl number in this limit. Thus we are of the opinion that the assertion of Dewey and Gross is most likely incorrect and that the true behavior of r for $\beta \to \infty$ will be revealed only by numerical calculation, or by analysis.

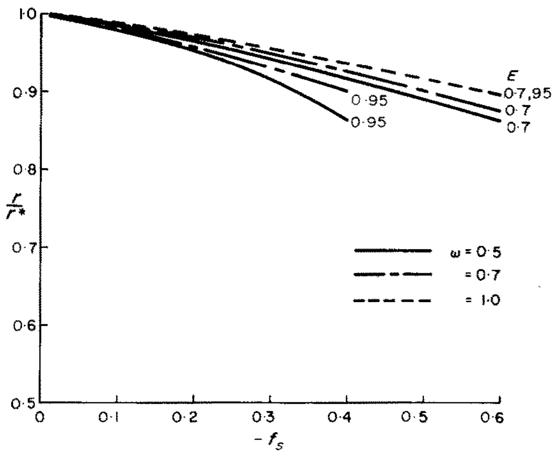

FIG. 6. Effect of viscosity law exponent ω on recovery factor. $Pr = 0{\cdot}740$, $\beta = 0$.

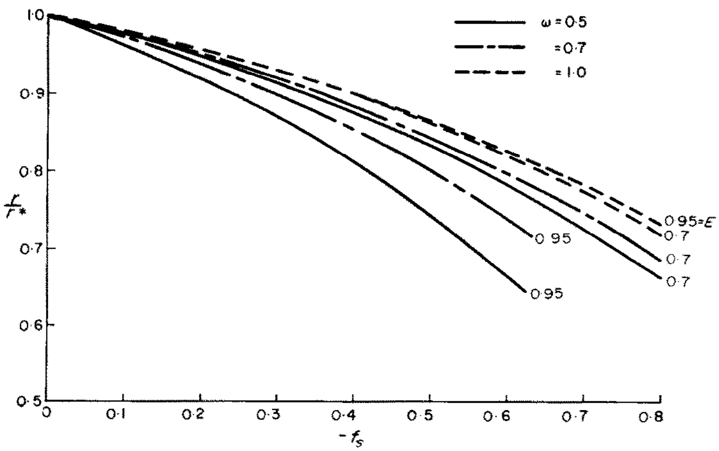

FIG. 7. Effect of viscosity law exponent ω on recovery factor. $Pr = 0{\cdot}740$, $\beta = 1{\cdot}0$.

Effect of viscosity law exponent ω

Figures 6 and 7 show the effect of ω on r for two particular values of β, $\beta = 0$ and $\beta = 1{\cdot}0$. Following usual practice the values of r are first normalized by the zero injection values r^* in order to remove the major effects of gas properties. Some property dependence remains and it can be seen that the effectiveness of injection in reducing the recovery factor increases with decreasing ω. The reason why normalization with r^* is not too successful in removing the property dependence is that the adiabatic wall enthalpy, and hence the enthalpy ratio across the boundary layer, is a function of the injection rate. We also note that the effectiveness is greater for $\beta = 1{\cdot}0$ as compared with $\beta = 0$. This trend is contrary to that observed for such quantities as the wall shear stress and heat flux. Though surprising, it can be understood by recognizing the similarity between highly blown and separating boundary layers; the latter have already been shown to be characterized by high recovery factors. Hence the more rapid decrease in wall shear stress with blowing for $\beta = 0$ is not accompanied by a similar behavior of recovery factor.

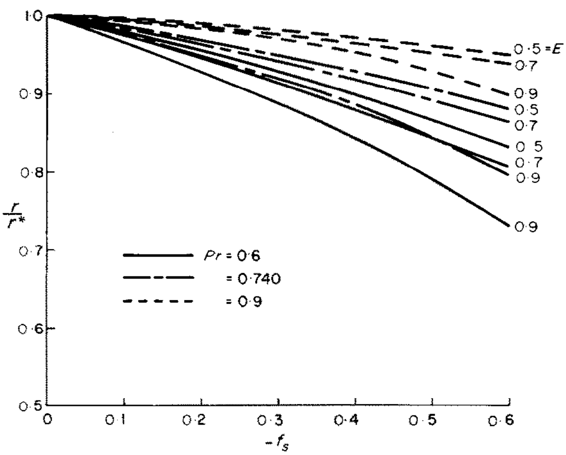

FIG. 8. Effect of Prandtl number on recovery factor. $\omega = 0{\cdot}5$; $\beta = 0$.

Effect of Prandtl number Pr

In this section of the discussion, the concern is not with the dependence of r on Pr, but rather with the effectiveness of injection in reducing r, as a function of Pr. Figures 8 and 9 show the effect of Pr for two particular values of β, $\beta = 0$ and $\beta = 1{\cdot}0$. The most extreme viscosity law, i.e. $\omega = 0{\cdot}5$ was chosen for this comparison as it yields the most pronounced effect. The increase of effectiveness of injection with decreasing Prandtl number is clearly displayed, and as was the case for ω, is more pronounced for the higher value of β.

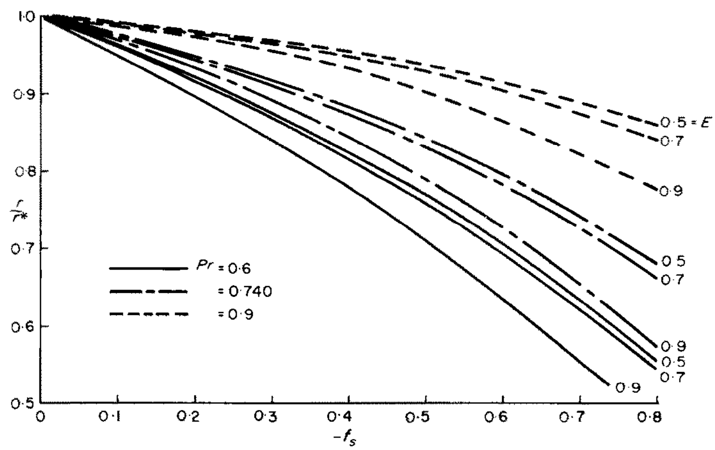

FIG. 9. Effect of Prandtl number on recovery factor. $\omega = 0{\cdot}5$; $\beta = 1{\cdot}0$.

Effect of energy parameter E

The parameter $E = u_e^2/2H_e$ is a measure of the kinetic energy available for conversion into thermal energy within the boundary layer. For flow over a flat plate E may be expressed in terms of the free stream Mach number, but, in so doing a value of the adiabatic exponent γ must be introduced. For hypersonic flow over a cone Newtonian flow theory yields $E = \cos^2\theta_c$, where θ_c is the cone half angle. We will, for convenience, refer to the effects of E as Mach number effects in accord with usual practice. The effect of Mach number is shown in all of Figs. 5–10, where it is seen that the deviation from unity increases with increasing E. The deviation becomes more pronounced at high injection rates; and as Fig. 10 shows, the behavior with suction is interesting due to the fact that $r \to 1$ as $f_s \to \infty$.

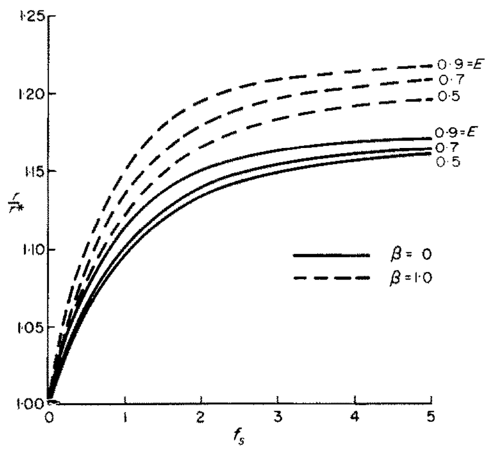

FIG. 10. Effect of suction on recovery factor. $Pr = 0{\cdot}740$: $\omega = 0{\cdot}5$.

Effect of injection rate $-f_s$

The effect of injection in reducing the recovery factor is best seen in Figs. 6–9. The effect of suction increasing the recovery factor is shown in Fig. 10 where, in addition, r/r^* exhibits an asymptotic behavior as r approaches unity. These results may be explained as follows. For $Pr < 1$, the zero injection adiabatic wall

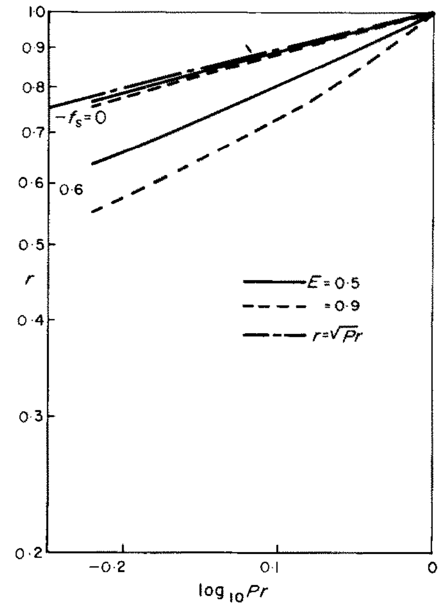

FIG. 11. Variation of recovery factor with Prandtl number. $\omega = 0{\cdot}5$: $\beta = 0$.

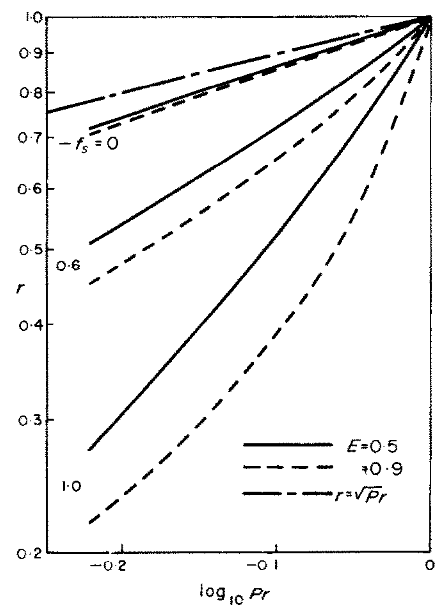

FIG. 12. Variation of recovery factor with Prandtl number. $\omega = 0{\cdot}5$: $\beta = 1{\cdot}0$.

enthalpy is less than the free stream total enthalpy. Injection thus adds fluid with a total enthalpy less than the average of the fluid in the boundary layer and a cooling effect results. Suction removes the lowest enthalpy fluid from the boundary layer and a higher wall enthalpy results. So far attention has been restricted to fluids with $Pr < 1$; for $Pr > 1$ the zero injection adiabatic wall enthalpy is greater than the free stream total enthalpy. Injection thus adds fluid with total enthalpy greater than the average of the fluid in the boundary layer; as a somewhat unexpected consequence, the wall enthalpy must increase. Examination of Figs. 11 and 12, where r is plotted against Pr, indeed does suggest the above-mentioned behavior for $Pr > 1$. A few solutions for $Pr > 1$ were in fact obtained, and they confirm our reasoning.

Recovery factor as a function of Prandtl number

Here attention is focused on the variation of r with Pr for a fixed injection rate. Figures 11 and 12 show this variation for two particular values of β, $\beta = 0$ and $\beta = 1{\cdot}0$ respectively. For zero pressure gradient constant property flows over an impermeable wall, Eckert [1] showed that $r = \sqrt{Pr}$ was an excellent approximation in the vicinity of $Pr = 1$; this result is also shown in the figures. In Fig. 11 it can be seen that the effect of increasing E is to yield recovery factors less than those predicted by the Eckert relation. This trend could, of course, also have been predicted from the previously mentioned effect of E on r and the fact that for $Pr = 1$, $r = 1$. In comparing the zero injection curves in Figs. 11 and 12, the previously mentioned effect of β on r is again exhibited in that the deviation from the Eckert result is greater for $\beta = 1{\cdot}0$. In addition, it is clear that the Eckert result would be obtained for values of β somewhat less than zero. Of course, in the presence of suction or injection, the Eckert result has no meaning, as demonstrated by the figures.

Correlation of the Mach number dependence

Following Gross *et al.* [14] and Simon *et al.* [5], an attempt was made to correlate the Mach number dependence by plotting against a modified injection parameter, $-f_s/\sqrt{C_r}$, where C_r is the density–viscosity product ratio evaluated at the Eckert reference enthalpy, equation (5). Figure 13 shows the results for the most extreme case, i.e. $\omega = 0{\cdot}5$. The correlation is adequate but could be improved. Figure 14 illustrates the

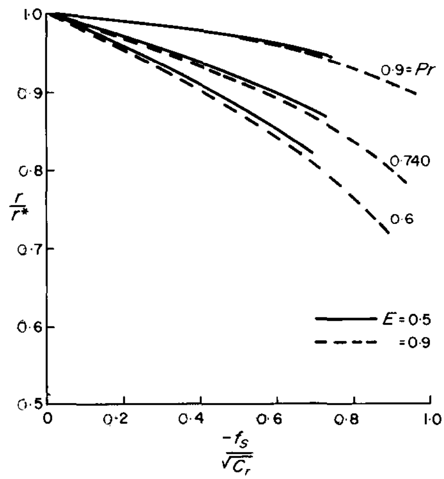

FIG. 13. Correlation of recovery factor data using C evaluated at the Eckert reference enthalpy. $\omega = 0{\cdot}5$: $\beta = 0$.

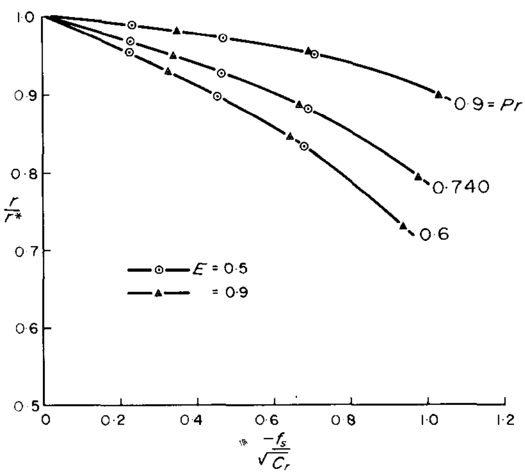

FIG. 14. Correlation of recovery factor data using C evaluated at the surface. $\omega = 0{\cdot}5$; $\beta = 0$.

correlation obtained if the data are plotted against $-f_s/\sqrt{C_s}$, i.e. with the density–viscosity product simply evaluated at the wall; the result is excellent. The superiority of the latter correlation suggests its use for foreign gas injection as well i.e. for data of [14] and [15].

Comparison with real gas data

Selected data for recovery factors in air, N_2 and CO_2 flows are shown plotted in Fig. 15. The recovery factors are normalized by $(Pr_s/0{\cdot}740)^{0{\cdot}5}$ to demonstrate that for a given flow situation the recovery factor is primarily a function of the flow Prandtl number. It should be noted that the surface value of the Prandtl number rather than the frequently suggested reference enthalpy value gives quite satisfactory correlation with model gas data for $\omega = 0{\cdot}5$ and constant Prandtl number. No significant improvement in the correlation could be found when reference enthalpy Prandtl numbers were tried and therefore use of the more conveniently computed wall values are suggested.

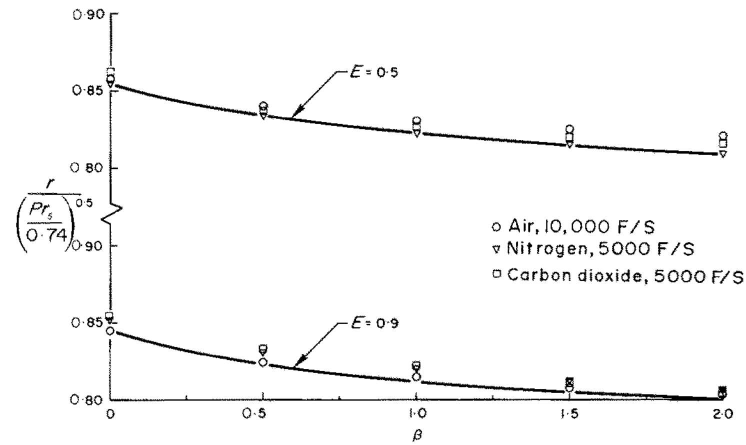

FIG. 15. Comparison of real gas data with the $\omega = 0{\cdot}5$ model gas predictions. $Pr = 0{\cdot}740$.

Comparison with experimental data

Experimental data suitable for verifying the results of this study do not appear in the open literature. However, one interesting observation can be made. Eckert [1], presenting results from a previous study [16], shows that for subsonic flow over a cylinder, the recovery factors do appear to start from a minimum value at the forward stagnation point. This result is in accord with the trend of r with β exhibited by the self-similar solutions.

CONCLUSIONS

1. The effect of the pressure gradient parameter β on the recovery factor is comparatively weak; r decreases monotonically with increasing β and the effect is most pronounced at high injection rates. With suction r tends to unity and thereby the effect is suppressed.

2. The effectiveness of injection in reducing the recovery factor (for $Pr < 1$) increases with decreasing ω.

3. For both injection and suction the deviation of r from unity increases with increasing Mach number; this deviation becomes more pronounced at high injection rates.

4. The Mach number dependence is better correlated if r is plotted versus $-f_s/\sqrt{C_s}$, rather than the frequently employed $-f_s/\sqrt{C_r}$.

5. For a given flow problem the real gas recovery factor may be correlated with the model gas ($\omega = 0{\cdot}5$, $Pr = 0{\cdot}740$) recovery factor when they are scaled by the square root of the Prandtl number evaluated at the surface.

6. Since experimental data for situations other than the impermeable flat plate are essentially nonexistent, the results presented here are the best available for engineering design purposes.

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L'EFFET DU TRANSFERT MASSIQUE SUR LES FACTEURS THERMIQUES PARIÉTAUX DANS DES ÉCOULEMENTS À COUCHE LIMITE LAMINAIRE

Résumé—On a obtenu dans une étude paramètrique des solutions numériques exactes pour des écoulements à couche limite adiabatique et à similarité. Les paramètres du problème sont: le gradient de pression (β = séparation à 2), l'énergie cinétique d'écoulement ($E = u_e^2/2He$ = 0,5, 0,7, 0,9, 0,95) et le flux massique pariétal ($-f_s$ variant de -10 à 1). De façon à montrer clairement l'effet des propriétés du gaz sur l'enthalpie de la paroi adiabatique et le facteur thermique pariétal $r = (h_{ad} - h_e)/(u_e^2/2)$, l'attention est attirée en premier sur des gaz à nombre de Prandtl constant (Pr = 0,6, 0,74, 0,9), une densité inversement proportionnelle à l'enthalpie statique et une loi puissance pour la viscosité ($\mu \propto h^{\omega}$, ω = 0,5, 0,7, 1). L'application de ce modèle d'écoulements gazeux aux écoulements de gaz réels est démontrée dans les problèmes pratiques par les expériences d'inclusion d'air, de N_2 et de CO_2 qui s'accordent avec les résultats du gaz théorique. On a trouvé que le facteur thermique pariétal décroît de façon monotone lorsque β augmente, l'effet étant plus prononcé à de hauts débits d'injection. De même, l'efficacité de l'injection pour réduire le facteur thermique pariétal croît lorsque ω diminue et l'écart de r à l'unité croît avec le nombre de Mach. A partir d'une normalisation en $Pr_s^{\frac{1}{2}}$, les facteurs thermiques pariétaux pour des gaz réels recouvrent les résultats du gaz théorique pour ω = 0,5. Les résultats présentés trouvent leur utilité principale dans le calcul pratique du refroidissement par ablation et par transpiration.

DIE AUSWIRKUNG DES MASSENTRANSPORTS AUF DEN RÜCKGEWINNFAKTOR IN LAMINAREN GRENZSCHICHTSTRÖMUNGEN

Zusammenfassung—Es wurden exakte numerische Lösungen für adiabate, ähnliche Grenzschichtströmungen in einer zusammenfassenden Parameteruntersuchung ermittelt. Die Parameter des Problems sind: Druckgradient (β = Ablösung bis 2,0), kinetische Strömungsenergie ($E = u_e^2/2H_e$ = 0,5; 0,7; 0,9; 0,95) und der Massenstrom durch die Oberfläche ($-f_s = -10{,}0$ bis 1,0). Um den Einfluss der Gaseigenschaften auf die adiabate Wandenthalpie und den Rückgewinnfaktor $r = (h_{ad} - h_e)/(u_e^2/2)$ klar aufzuzeigen, wurde die Aufmerksamkeit zunächst auf Modellgase mit konstanten Prandtl-Zahlen (Pr = 0,6; 0,74; 0,9), mit einer Dichte proportional dem Kehrwert der statischen Enthalpie und mit einem Potenzansatz für die Viskosität ($\mu \sim h^{\omega}$; ω = 0,5; 0,7; 1,0) gerichtet. Die Gültigkeit der Modellgasströmungen für reale Gasströmungen bei Ingenieuraufgaben wurden durch Daten für Luft, N_2 und CO_2, die den Ergebnissen der Modellgasrechnung sehr gut gehorchen, aufgezeigt. Es ergab sich, dass der Rückgewinnfaktor monoton mit zunehmendem β abnimmt, wobei dies bei hoher Massenzufuhr am ausgeprägtesten ist. Ebenso wächst die Wirkung der Massenzufuhr auf die Verringerung des Rückgewinnfaktors mit abnehmendem ω. Die Abweichungen des Rückgewinnfaktors von eins nehmen mit wachsender Machzahl zu. Bei Verhältnisbildung mit $Pr^{\frac{1}{2}}$ stimmen die Rückgewinnfaktoren für reale Gase gut mit den Modellgasrechnungen für $\omega = 0{,}5$ überein. Die vorliegenden Ergebnisse finden ihre hauptsächliche Anwendung bei der Berechnung von Ablations- und Transpirationskühlungen.

ВЛИЯНИЕ МАССООБМЕНА НА КОЭФФИЦИЕНТЫ ВОССТАНОВЛЕНИЯ В ЛАМИНАРНОМ ПОГРАНИЧНОМ СЛОЕ

Аннотация—Параметрическим методом получены точные численные решения для адиабатических пограничных слоев в автомодельной области. Параметры задачи следующие: градиент давления (β = отрыв до 2,0), кинетическая энергия потока ($E = u_e^2/2H_e$ = 0,5; 0,7; 0,9; 0,95) пр скорость массового потока на стенке. С целью уточнения влияния свойств газа на энтальпию адиабатической стенки и коэффициент восстановления рассматривались, главным образом, газы с постоянным числом Прандтля, у которых плотность обратно пропорциональна статической энтальпии, а вязкость подчиняется степенному закону. Справедливость моделирования для изучения течения реальных газов в инженерных задачах подкреплена хорошим соответствием результатов, полученных для воздуха, азота и модельного газа. Обнаружено, что коэффициент восстановления монотонно снижается по мере увеличения β, причем этот эффект наиболее существенно сказывается при больших вдувах. Эффективность вдува при уменьшении коэффициента восстановления растет с уменьшением ω, а отклонение r от 1 увеличивается с возрастанием числа Маха. Введением поправки в виде $Pr_s^{1/2}$ достигается хорошее соответствие реальных газов с модельным при ω = 0,5. Полученные результаты могут найти прямое применение в инженерных расчетах абляционного и испарительного охлаждения.